International Journal of Mathematics and Physical Sciences Research ISSN 2348-5736 (Online)
Vol. 3, Issue 1, pp: (166-177), Month: April 2015 - September 2015, Available at: www.researchpublish.com

MODELLING THE GROWTH OF INTERNET BROADBAND IN RWANDA

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Abstract: **This research aims to determine a model that would predict the future broadband growth in Rwanda over a period of time. More often Organizations/Companies make predictions basing on the available technology based on the available data. But these data predictions more than often are influenced by disruptive technologies. This research was to therefore model the growth of internet broadband in Rwanda, with a view of technological changes and be able to predict the broadband growth irrespective of future technologies. This research targeted and collected data from all internet licensed operators in Rwanda. This model will help planners especially operators to determine how long a certain technology is likely to last, its output and be able to avoid unnecessary stocks that lead to losses due to obsolete technologies which are overtaken by new technologies. The Gompertz curve was for long been used only to actuaries but more recently, it has been used by many authors as growth curve both for biological and for economic purposes. The intention therefore was to determine the constants, Inflection, Straight line form of equation, Growth rate, Maximum growth rate, Relative growth rate as function, Relative growth rate as function of size. The research found out that the data fits the Gompertz model at 95% of confidence level. The parameters, Lambda λ (inflection growth rate), mu,μ(relative growth rate, A(Maximum growth rate) were computed to able to determine the model and to predict the future internet broadband requirement in Rwanda at any time (t).**

Keywords: **Inflection, Relative growth rate, Maximum growth rate, Gompertz model and Internet broadband.**

I. INTRODUCTION

1.1 Problem Statement:

Is the growth rate in the past a good indicator of growth in the future? PROBABLY NOT especially for internet firms. In this STUDY I will consider how good historical growth is as a predictor of future growth for all firms, and why the changing size and volatile businesses of internet firms can undercut growth projections. Poor modeling or even no modelling at all may lead slow growth, lower valuations, slowing acquisition of customers due to market saturation, declining lifetime value of new customers, decreasing participation of ecosystem partners (developers or channel resellers), disruption technologies from new entrants and ultimately in the loss of market relevance. Consequently, knowing when to transition is critical. A final barometer of impending slowdown is the loss of key talent from sales, presales, or engineering. When the moment is right, companies should have good models to be able to avoid said pitfalls. First, some companies select the wrong market or product offering for their second act. This failure can be attributed to insufficient diligence in assessing the new market or not having the right capabilities in-house to design and build that next major offering. Companies/Organizations can also under invest in the resources or budget required due to prediction inabilities.

1.2 Objectives:

1.2.1 General Objectives:

This project aims at getting a model that will be able to predict the future broadband requirements in Rwanda and how companies, Government and organizations can mitigate loses originating from disruptive technologies and be able to save money by purchasing internet bandwidth a head of time.

1.2.2 Specific Objectives:

1. To develop a growth model of internet broadband in Rwanda with changing technologies

2. To see impact of increased supply and demand of internet bandwidth on prices.

3. To predict the broadband ($y(t)$) at any time (t)

1.3 Research Questions:

1. Does changing technologies affect increase in internet broadband in Rwanda?

2. Does increase in supply and demand of internet broadband affect prices?

3. Given a model, is it possible to determine internet broadband y(t) at a future time (t) ?

1.4 Justification of the study:

The research findings will help the policy makers and operators to effectively predict growth of internet in Rwanda and how long a certain technology can last.

1.5 Scope of the study:

The study will cover all internet traffic generated in Rwanda through all internet licensed operators

II. METHODOLOGY

If it is possible to predict with a high degree of certainty that, there is money to be made; more important, such a situation would reflect a rather basic malfunctioning of the market mechanism. In practice, however, investments in portfolio involve risk, and predictability becomes a statistical concept. During this study I intend to offer the most complete, up-to-date and accurate source for Rwanda broadband statistics and estimates. In order to do this, I will collect quarterly statistics from all internet providers and telecomm operator from operators themselves and from the Rwanda Utility Regulatory Agency. Many operators now publish quarterly numbers as part of their regular reporting cycle.

2.1 Data collection:

Many operators continue to release annual reports as opposed to quarterly ones. Some also choose to aggregate subscriber trends into overall totals, avoiding break-downs by technology. In these cases, I will conservatively estimate broadband take-up. Key sources for such estimated totals typically include prior and partial reports by the operators themselves. Rwanda Utility Regulatory Agency (RURA) also frequently report broadband statistics.

I will continue to maintain close correspondence with broadband operators, RURA and industry organizations like Rwanda Development Board, Ministry of Information and Communication (MYICT) and ICT chamber in the private sector in order to avoid ambiguities and also so as to minimize the number of restatements.

2.2 Forecast of broadband Data:

I will use the **Gompertz Curve** to forecast Rwandan broadband growth. According to point- topic (research organization), the Gompertz Curve has been used for modeling consumer behavior in a number of instances for at least the last 50 years. It has proven to be an accurate and powerful tool for predicting the diffusion of technology products through a consumer universe. Gomperzt model is given as;

$$y(t) = A.\exp\left[-\exp\left(\frac{\mu.\exp(1)}{A}(\lambda - t) + 1\right)\right]$$

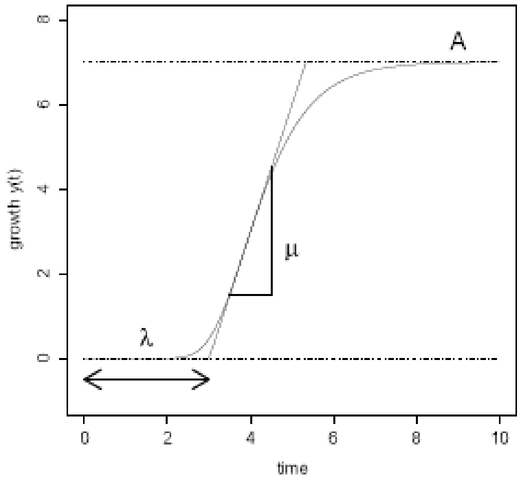

Figure 2.1: Typical parameters derived from growth curves: length of lag phase λ, growth rate represented by the maximum slope μ and the maximum internet growth A.

The study, therefore, fitted growth data which show a point of inflection in the early part of the growth cycle, approximately 34 to 40 percent of the total growth has been realized, we may use the Gompertz curve with the expectation that the approximation to the data will be good. The intention therefore is to determine the constants, Inflection, Straight line form of equation,

Growth rate, Maximum growth rate, Relative growth rate as function, Relative growth rate as function of size

Description of Equation 3.1 is as follows:

The function calculates the values of the Gompertz growth curve for given time points.

Usage gompertz (time, A, mu, lambda, addpar = NULL) .

Time (t) : Time points (x-axes) for which the function values will be returned.

A: Maximum of the curve. If a vector is provided only the first entry is used.

Mu (μ): Maximum slope. If a vector is provided only the first entry is used.

Lambda (λ) : Lag-phase. If a vector is provided only the first entry is used.

addpar : Additional parameters have no effect in this type of model. They belong to the standard model description in grofit and are initialized as addpar=NULL in the function header.

The Gompertz curve was for long been used only to actuaries but more recently, it has been used by many authors as growth curve both for biological and for economic purposes.

I will also forecast using the following inputs:

1. **Broadband subscribers -** every quarter, this historical data is used as the basis for our forecasts.
2. **Population forecasts** – I will use population forecasts published by the National institute of statistics of Rwanda.
3. **Household forecasts –** I will use estimates of the number of people per household to forecast growth in households

We then calculate the following variables for each of the markets covered:

1. **Coverage** - coverage defines the total addressable audience and is therefore set as the upper limit for broadband subscriptions. I forecast coverage growth over time.
2. **Inception date** - how many quarters since the technology was adopted in that market? This will affect the growth rate.
3. **Growth factor** - the speed of adoption (diffusion) of fixed broadband in a particular market. This is an indicator of how accessible, both economically (consumers can afford it) and in terms of availability (the service is present for a consumer in a particular place), fixed broadband is. This will vary from market to market.

With Gompertz model (recommended by point-topic), I will forecast the future course of a partially complete substitution. Using regression methods, the appropriate model is fit to the historical data to obtain best-fit estimates of the parameters **a** and **b**.

2.3 Drivers of Bandwidth Demand:

The quantity of bandwidth being demanded is increasing. This thesis will look at the following drivers contributing to bandwidth demand. Other drivers may exist but can usually be included in one of the following categories:

1. Rwanda Broadband policy
2. Rwanda online, smart Rwanda
3. Rwanda Regulatory and enabling environment

III. RESEARCH FINDINGS AND DISCUSSION

3.1 Introduction:

In this chapter, i have described how data analysis was done and the findings have been presented. The main results are presented and finally the model is fitted and is used to respond to the research data of this study and address the main objectives. The general objective of this project is to evaluate and fit a model that would predict the future bandwidth requirements in Rwanda and corresponding prices. In this study I used the secondary data with the main source of data being the regulator who corrects data from all operators on a quarterly basis indicating total number of megabytes used and price per megabyte.

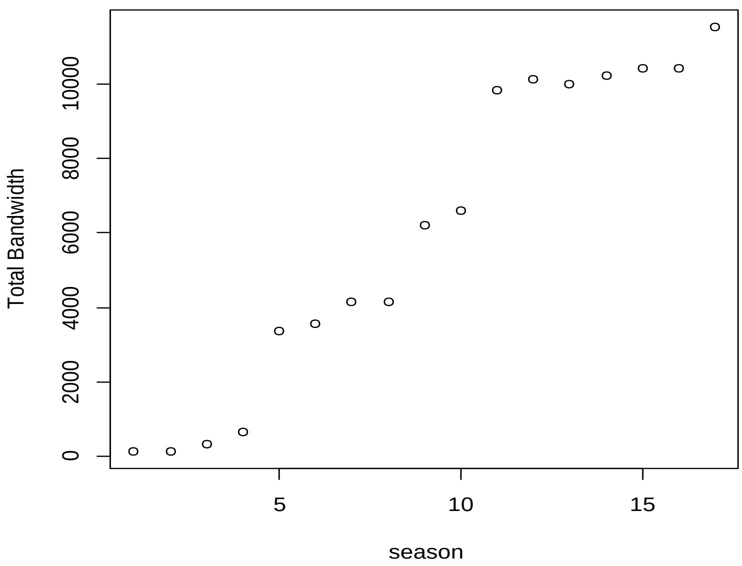

Figure 3.1: Scatter Plot of internet megabytes used

From the scatter plot, it is clear that the data follows an S curve and Gompertz function would probably better fit the model.

Table 3.1: Quarterly bandwidth consumption in MBPS

The data collected from Rwanda Utility Regulatory Agency (RURA) was recorded quarterly from 2007 to 2014; the available data in table1 were only for the last quarter of each year.

Quarterly season (2007-2014)	TOTAL Bandwidth in MBPS
1	156.6
2	156.6
3	350.8
4	654.8
5	3381
6	3582
7	4164

8	4164
9	6220
10	6600
11	9829
12	10119
13	9994
14	10222
15	10412
16	10412
17	11514

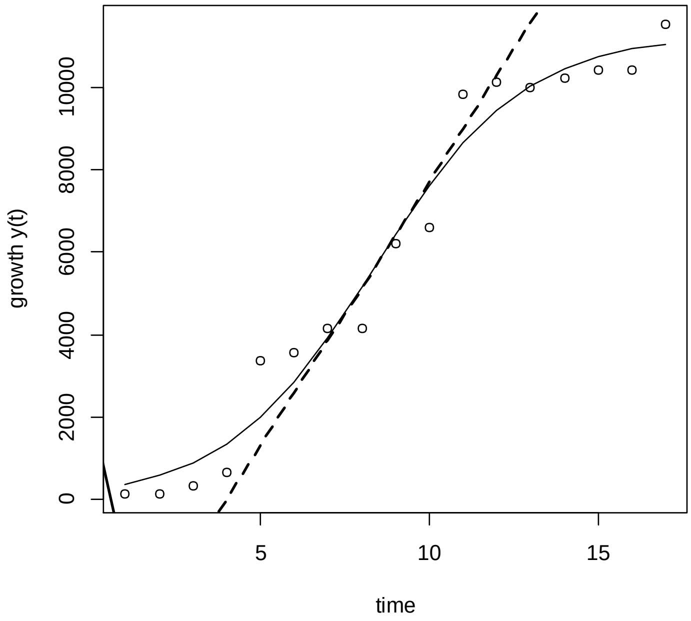

Figure 3.2: Original data of Internet Broadband

3.2 The Model:

Running grofit command in R software it gives the output bellow which was used to fit the model and predict the future internet bandwidth.

Mu. Model lambda. Model A. model integral. Model stdmu.model stdlambda.model

1279.785 3.977929 11261.5 96736.85 142.865 0.5201849

stdA.model ci90.mu.model.lo ci90.mu.model.up ci90.lambda.model.lo

536.4476 1044.772 1514.798 3.122225

ci90.lambda.model.up ci90.A.model.lo ci90.A.model.up ci95.mu.model.lo

4.833634 10379.04 12143.96 999.7694

ci95.mu.model.up ci95.lambda.model.lo ci95.lambda.model.up ci95.A.model.lo

1559.8 2.958367 4.997492 10210.06

ci95.A.model.up 12312.94

The output of the model above is defined as follows:

Mu (μ) =1279.785

Lambda (λ) =3.977929

A=11261.5

$$y(t)=11261.5.\exp\left[-\exp\left(\frac{1279.785.\exp(1)}{11261.5}(3.977929-t)+1\right)\right]$$

The outputs of the model above are defined as follows:

mu.model	max. slope
lambda.model	lag-phase
A.para	maximum growth
Integral.model	integral
stdmu.model	standard deviation (cross validation)
stdlambda.model	standard deviation (cross validation)
stdA.model	standard deviation A (cross validation)
ci90.mu.model.lo	90% CI lower boundary
ci90.mu.model.up	90% CI interval upper boundary
ci90.lambda.model.lo	90% CI interval lower boundary
ci90.lambda.model.up	90% CI interval upper boundary
ci90.A.model.lo	90% CI interval lower boundary A
ci90.A.model.up	90% CI interval upper boundary A
ci95.mu.model.lo	95% CI interval lower boundary
ci95.mu.model.up	95% CI interval upper boundary
ci95.lambda.model.lo	95% CI interval lower boundary
ci95.lambda.model.up	95% CI interval upper boundary
ci95.A.model.lo	95% CI interval lower boundary A
ci95.A.model.up	95% CI interval upper boundary A

The study used the above value to make projections at any time (t). The confidence interval at 90% for mu, lambda and A were as follows, Mu is between 1044.772 and 1514.798, lambda is between 3,122225 and 4.833634, A is between 10379.04 and 12143.96. With at 95%, the confidence interval for mu, lambda and A is as follows, mu will lie between 999.7694 and 1559.8, lambda lies between 2.958367 and 4.997492 and A lies between 10210.06 and 12312.94. Therefore the model fits best at significant level of 90% and 95%.

Table 3.2: Predicted internet bandwidth

Time	Prediction	Time	Prediction
t=17	K=(1279.785*exp(1)/11261.5)*(3.977929-t) K M=K+1 y17=11261.5*exp(-exp(M)) y17 t=17 K=(1279.785*exp(1)/11261.5)*(3.977929-t) K M=K+1 y17=11261.5*exp(-exp(M)) t=17 K=(1279.785*exp(1)/11261.5)*(3.977929-t) K M=K+1 y17=11261.5*exp(-exp(M))	t=21	K=(1279.785*exp(1)/11261.5)*(3.977929-t) K M=K+1 y21=11261.5*exp(-exp(M))
		t=21	K=(1279.785*exp(1)/11261.5)*(3.977929-t) K M=K+1 y21=11261.5*exp(-exp(M))
		t=22	K=(1279.785*exp(1)/11261.5)*(3.977929-t) K M=K+1 y22=11261.5*exp(-exp(M))
		t=23	K=(1279.785*exp(1)/11261.5)*(3.977929-t) K M=K+1 y23=11261.5*exp(-exp(M))
t=18	K=(1279.785*exp(1)/11261.5)*(3.977929-t) K M=K+1 y18=11261.5*exp(-exp(M))	t=24	K=(1279.785*exp(1)/11261.5)*(3.977929-t) K M=K+1 y24=11261.5*exp(-exp(M))
t=19	K=(1279.785*exp(1)/11261.5)*(3.977929-t) K M=K+1 y19=11261.5*exp(-exp(M))	t=25	K=(1279.785*exp(1)/11261.5)*(3.977929-t) K M=K+1 y25=11261.5*exp(-exp(M))
t=20	K=(1279.785*exp(1)/11261.5)*(3.977929-t) K M=K+1 y20=11261.5*exp(-exp(M))		

Table 3.3: Data and predicted data of internet bandwidth:

Table 3.3 present the data collected from Rwanda Utility Regulatory Agency (Season 1 up to season17) and predicted data from season 18 up to season 25.

Season	Bandwidth(MBPS)
1	156.56
2	156.56
3	350.765
4	654.768
5	3381
6	3582
7	4164
8	4164
9	6220
10	6600
11	9829
12	10119
13	9994
14	10222
15	10412
16	10412
17	11514
18	10866.16
19	10969.85
20	11046.61
21	11103.32
22	11145.14
23	11175.94
24	11198.62
25	11215.29

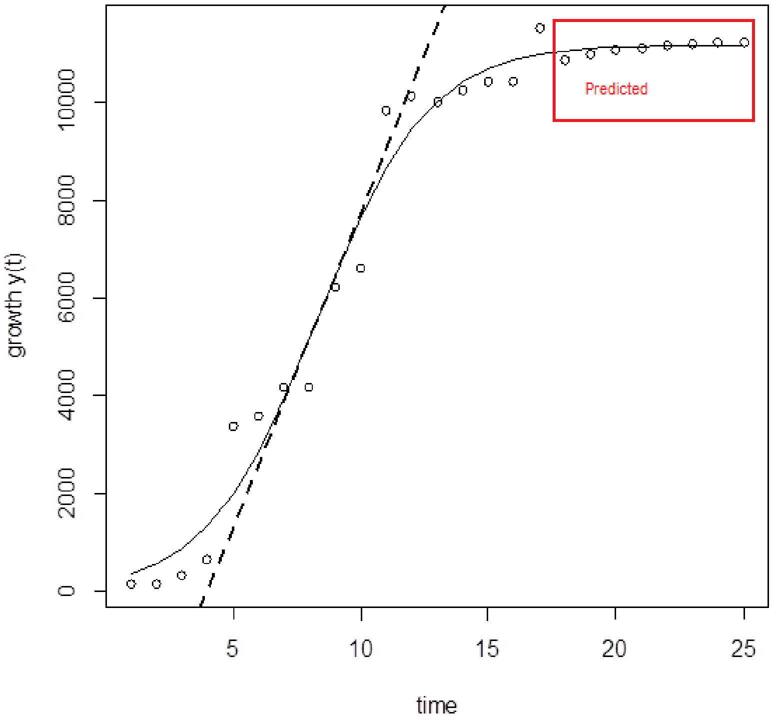

Figure 3.3: Plot of data and Predicted data Internet bandwidth

Table 3.4: Internet prices per season from 2007 to 2014

Internet prices per season	
season1	**price.frw.**
1	2453215
2	2453215
3	2324325
4	1900000
5	1750000
6	1540000
7	1250000
8	900000
9	600568
10	520568
11	364682
12	318182
13	311844
14	292456
15	258083
16	249083
17	245000

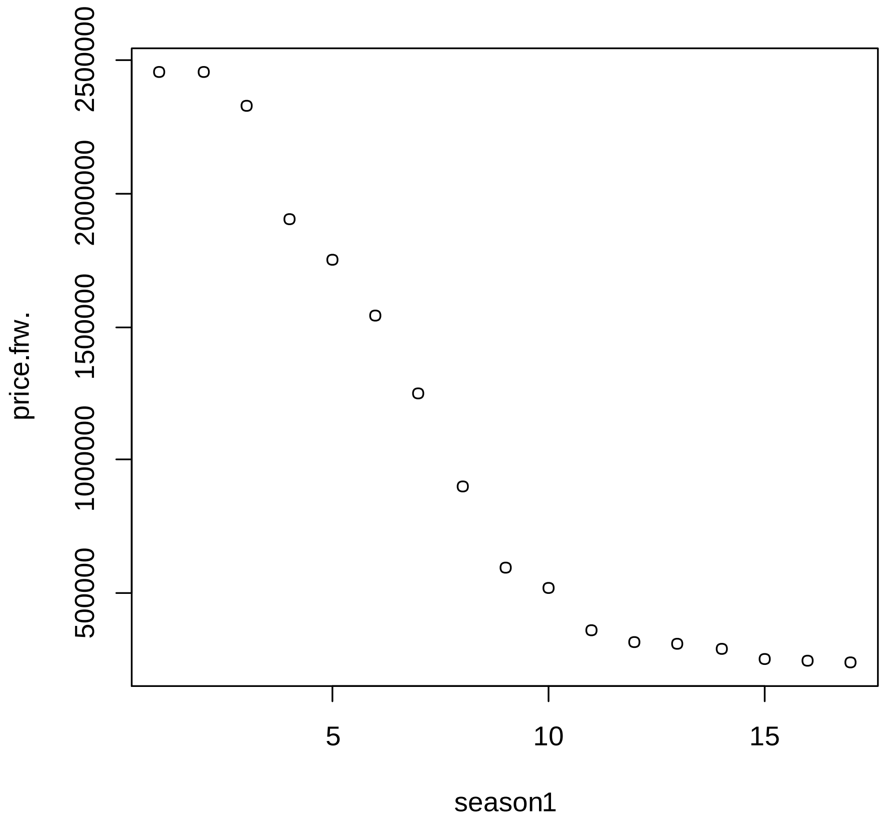

Figure 3.4: Scatter Plot of internet prices

Plotting the data in table3.3 using R Software its shows the spline below in the figure3.5

mu.spline lambda.spline A.spline integral.spline

7908.373 -309.2048 2453215 16383172

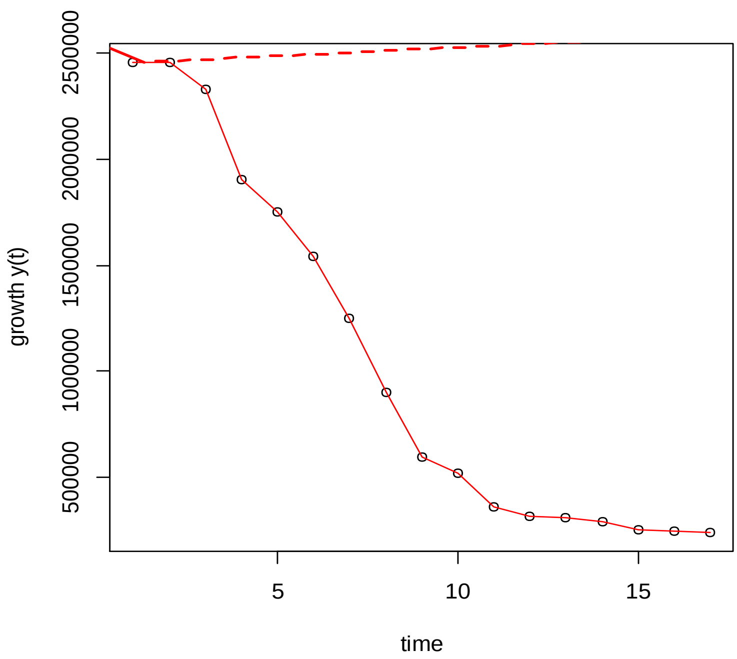

Figure 3.5: Plot of splined internet prices

By mere spinning of the internet bandwidth prices, the parameters mu (μ) is 7908.373, lambda (λ) is -309.2048, while A is 2453215. From supply and demand in comparison to prices, it is evident that as demand and consumption of internet bandwidth increases, the price decreases. For the past seventeen seasons, the increase in consumption of internet bandwidth has resulted in 77.7% reduction in price unit cost (megabyte per second) of internet bandwidth. However, this percentage may not continue to increase as prices will not come to zero will rather remain asymptotically to a small value and this requires constant monitoring.

IV. SUMMARY, CONCLUSION AND RECOMMENDATIONS

4.1 Introduction:

This chapter will try to summarize results found in my study, give conclusion on the findings and finally I will come out with recommendations.

To achieve its goal, this chapter will be divided into three subtitles: Summary, Conclusion and recommendations.

4.2 Summary:

The purpose of my project is to develop a model for the growth of internet broadband in Rwanda and be able to predict the future of internet requirements in Rwanda. The reason for my research is to provide necessary information to policy makers and internet providers as well as users in order to plan ahead of time.

The project is composed of five chapters, each of them dealing with different aspects of modelling the growth of internet bandwidth in Rwanda. Chapter One is introductory and deals with the background of the project, statement of the problem, objectives, research questions, justification and scope of the research.

Chapter Two consists of the literature review of the modelling of non parametric regression. The chapter defines basic terminology used in the project and explores the theoretical review of different types of modelling techniques especially using Gompertz model.

Chapter Three provided the methodology and framework of my project. This chapter describes all the methods and instruments used to collect the information I need as well as the procedure of the analysis and interpretation of the information gathered.

Chapter Four concentrated on my research findings and discussion on results found after analysis. This chapter shows different phases, the lag phase (λ),the log phase(μ) and the constant phase (A). These phases give an indication and a course of action for the policy makers and investors on how the market is responding and what appropriate measures that can be taken. The model shows that A was 11261.5 mbps and this serves as an eye opener to the policy makers that something should be done to change trend.

Conclusions are drawn in Chapter Five. The main aim of the project is to develop a model and predict the growth of internet usage in Rwanda and see if there any relationship between increase in use of internet bandwidth and reduction price per unit price. It is therefore imperative that investors who are dealing in internet broadband and who purchase long term capacity do understand implications in order to get a return on their investments.

4.3 Conclusions:

The main objective of this project is to develop a model for the growth of internet broadband in Rwanda and be able to predict its future. However, from the findings, internet usage seems to have reached a constant stage and this should not be the case considering internet penetration in Rwanda. However, I used secondary data and there is need to do more research based on primary data and verify the authenticity of the data, nevertheless the government should do a lot more to keep an increasing rate among others;

(i) There is need to increase the overall ICT digital literacy among the population, including

(ii) the public sector employees, teachers, etc. and adapted local content applications in order to facilitate the use of ICT;

(iii) Rwanda has largely mainstreamed ICT policies in the last years to support and enhance the delivery of public and private services but this require deeper integration;

(iv) The Ministry should undertake sector and institutional reform; instil ICT being business oriented rather than supporting functions.

(v) ICT should continue beef up the efficiency of the public administration and private sector;

(vi) A call has been made for private sector increase ownership, turn current issues into private opportunities, need for innovative approaches, assimilate technology, mind-set change and, focus on new investment in Mobile Solutions, BPO, IT Security, Cloud and IT

(vii) Training, and E-Government; Engage the Private sector in ICT sustainable skills development program and helping academia to develop high quality employable skills;

(viii) In the long run, there an opportunity cost to pursue BPO-focused investments in Rwanda versus potentially high-end services like data analytics, software/app development, programming, etc.

Policy-makers need to support innovation, entrepreneurship and talent, through educational measures, fiscal incentives and industrial policy. Public-Private Partnerships (PPPs) can also transfer skills, capabilities and technologies,

I. By creating local ICT ecosystems with technology hubs and innovation incubators;

II. By supporting long-term innovation capacity through the enhancement of skills and knowledge; by empowering citizens through access to information and apps; or

III. By opening up new financing for start-up businesses. Featured Insight 5 explores how broadband is acting as an ‘accelerator’, driving change across all four major pillars of innovation – people, ideas, finance, as well as markets.

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